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VARIATSION PRINSIPDAN FOYDALANIB, MATEMATIK MODELLASHTIRISHNING AHAMIYATI.

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Annotatsiya: Ushbu maqolada matematik modellashtirishning variatsion prinsiplarga asoslangan metodlari yoritiladi. Lagranj funksionali, Eyler–Lagranj tenglamalari, minimal harakat prinsipi, energiya funksionalining ekstremali kabi nazariy tushunchalar modellar qurishda qanday qo'llanilishi ilmiy jihatdan tahlil qilinadi. Mexanika, fizik maydonlar, elastiklik nazariyasi va optimal boshqaruvga doir misollar orqali variatsion usulning amaliy ahamiyati ko'rsatib berilgan.

Kalit so'zlar: Variatsion prinsip, funksional, Eyler–Lagranj tenglamasi, minimal harakat prinsipi, energiya funksionali, optimal boshqaruv, minimal sirt, geodeziklar, Ritz usuli, Galerkin usuli, chekli elementlar metodi, Sobolev fazolari, matematik modellashtirish

Аннотация: В данной статье рассматриваются методы математического моделирования, основанные на вариационных принципах. Научно анализируются такие теоретические понятия, как функционал Лагранжа, уравнения Эйлера–Лагранжа, принцип наименьшего действия и экстремали энергетического функционала, а также способы их применения при построении математических моделей. На примерах из механики, теории физических полей, теории упругости и оптимального управления показано практическое значение вариационного подхода.

Ключевые слова: Вариационный принцип, функционал, уравнение Эйлера–Лагранжа, лагранжиан, принцип наименьшего действия, энергетический функционал, оптимальное управление, минимальная поверхность, геодезические, метод Ритца, метод Галёркина, метод конечных элементов (МКЭ), пространства Соболева, математическое моделирование.

Abstract: This article examines mathematical modeling methods based on variational principles. Theoretical concepts such as the Lagrange functional, Euler–Lagrange equations, the principle of least action, and extremals of energy functionals are scientifically analyzed in the context of constructing mathematical models. Examples from mechanics, physical field theory, elasticity theory, and optimal control demonstrate the practical significance of the variational approach.

Keywords: Variational principle, functional, Euler–Lagrange equation, Lagrangian, principle of least action, energy functional, optimal control, minimal surface, geodesics, Ritz method, Galerkin method, finite element method (FEM), Sobolev spaces, mathematical modeling.

Kirish: Variatsion prinsip tabiiy va texnik jarayonlarning fundamental matematik modeli sifatida keng qo‘llaniladi. Tabiatdagi ko‘plab jarayonlar **minimal energiya**, **minimal harakat**, **eng qisqa yo‘l** yoki **optimal konfiguratsiya** tamoyillariga asoslanadi. Shu sababli variatsion metodlar:

mexanika,
akustika,
elektromagnit maydonlar,
issiqlik va diffuziya jarayonlari,
elastiklik nazariyasi,
optimal boshqaruv,
kvant fizika
kabi ko‘plab sohalarda asosiy matematik metod sifatida ishlatiladi.

Variatsion prinsipning mathematik negizi **funksional ekstremumini topish** muammosiga borib taqaladi.

Variatsion prinsiplarning nazariy asoslari

Funksional va uning ekstremali

Variatsion masalaning mohiyati quyidagi funksionalni minimallashtirish yoki maksimallashtirishdan iborat:

$$J[y] = \int_a^b F(x, y, y')dx$$

Bu yerda:
y(x) – qidirilayotgan funksiya,
F– Lagranj funksiyasi,
J[y] – funksional.

Ekstremum sharti Eyler–Lagranj tenglamasiga olib keladi:

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

Bu tenglama variatsion masalalarning asosiy matematik modeli sifatida qaraladi.

Minimal harakat prinsipi (Hamilton–Lagranj prinsipi)

Klassik mexanikada tizimning haqiqiy trayektoriyasi **harakat funksionali**ning minimumi bilan aniqlanadi:

$$S[q] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$$

Bu yerda L=T−V – Lagrangian (kinetik va potensial energiyalar farqi).

Ekstremal shart:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q'}\right)-\frac{\partial L}{\partial q}=0$$

Natijada klassik mexanikaning asosiy tenglamalari hosil bo‘ladi.

Variatsion prinsip asosida matematik model qurish

Eng qisqa yo‘l masalasi (geodeziklar)

Funksional:

$$J[y]=\int_a^b \sqrt{1}+(y')^2dx$$

Eyler–Lagranj tenglamasi tekislikdagi to‘g‘ri chiziqni beradi.

Bu model optika, geometriya, fizikaviy jarayonlarda qo‘llanadi.

Yuzani minimallashtirish – minimal sirt modeli

Minimal sirt masalasi quyidagicha yoziladi:

$$J[z]=\int\int_\Omega \sqrt{1+z_x^2+z_y^2}\,dxdy$$

Natijada **minimal sirt tenglamasi** hosil bo‘ladi:

$$\frac{\partial}{\partial x}\left(\frac{z_x}{1+|\nabla z|^2}\right)+\frac{\partial}{\partial y}\left(\frac{z_y}{1+|\nabla z|^2}\right)=0$$

Bu model kapillyar sirtlar, plyonkalar va sirt tarangligi muammolarida qo‘llanadi.

Elastiklik nazariyasida energiya funksionali

Qattiq jismlar deformatsiyasida potensial energiyani minimallashtirish orqali model quriladi:

$$J[u]=\int_\Omega \Omega W(\varepsilon(u))\,d\Omega$$

Bu yerda:

W– deformatsiya energiyasi,

ε – deformatsiya tensori.

Eyler–Lagranj tenglamasi elastiklik nazariyasining asosiy differensial tenglamalariga olib keladi:

$$\nabla\cdot\sigma+f=0$$

Issiqlik va diffuziya jarayoni uchun variatsion formulirovka

Issiqlik tenglamasining variatsion ko‘rinishi:

$$J[u]=\int_0^T\int_\Omega\left(\frac{1}{2}|\nabla u|^2-fu\right)d\Omega dt$$

Bu funksionalning ekstremali Furye issiqlik tenglamasiga olib keladi:

$$u_t-\Delta u=f(x,t)$$

Optimal boshqaruv nazariyasida variatsion yondashuv

Optimal boshqaruvning asosiy funksionali:

$$J[u]=\int_0^T L(x(t),u(t),t)dt$$

Pontryagin maksimum prinsipi:

$$H(x, u, \lambda) = \max_{u} H$$

Bu metod orqali iqtisodiy modellar, populyatsiya boshqaruvi, energiyani minimallashtirish masalalari yechiladi.

Variatsion tenglamalarning sonli yechimi

Variatsion masalalarni yechishda quyidagi usullar qo‘llanadi:

Ritz metodi

Galerkin usuli

Chekli elementlar metodi (FEM)

Chekli farqlar variatsion analogi

Sobolev fazolari va Hilbert fazolarda yondashuv

FEM – zamonaviy muhandislik hisoblarining asosini tashkil qiladi.

Xulosa: Variatsion prinsip tabiiy jarayonlar va texnik tizimlarning fundamental matematik modeli bo‘lib, ko‘plab qonuniyatlarning umumiylashgan matematik ifodasini beradi. Bu prinsip asosida mexanika, elastiklik, issiqlik va diffuziya, optimal boshqaruv, maydon nazariyalari, uchun universal matematik modellar qurish mumkin.

Foydalanilgan adabiyotlar

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