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Modelling physical processes based on a three-dimensional wave–diffusion model

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Abstract. This paper investigates a three-dimensional wave-diffusion equation as a unified mathematical model for describing physical processes involving both wave propagation and energy dissipation. In contrast to the classical wave equation, the proposed model incorporates a damping term that accounts for diffusion and energy loss in real physical media. An analytical solution is obtained using the method of separation of variables, which reduces the problem to a spectral boundary-value problem for the Laplace operator and a damped second-order ordinary differential equation in time.

The resulting solutions describe exponentially decaying oscillations, where the damping coefficient governs the rate of energy dissipation and the spectral parameters determine the frequency characteristics of the system. The physical interpretation of the model is discussed in the context of acoustic wave attenuation, thermoelastic interactions, electromagnetic signal weakening, and other dissipative phenomena. The proposed approach provides a general analytical framework for modelling coupled wave-diffusion processes and can serve as a basis for further numerical simulations and applied studies in physics and engineering.

Keywords: wave-diffusion equation, damping effect, Laplace operator, analytical solution, mathematical modelling.

Introduction. Partial differential equations of mathematical physics play a fundamental role in the analysis and modelling of physical phenomena arising in science and engineering. Processes such as wave propagation, heat transfer, diffusion, and electromagnetic field evolution are commonly described using differential equations that capture both spatial and temporal variations.

The classical wave equation models the propagation of energy without losses. However, real physical media are characterized by resistance, friction, heat exchange, and other dissipative effects that lead to gradual energy decay over time. Consequently, there is a strong need for hybrid mathematical models that combine wave propagation with diffusion and damping mechanisms.

In this paper, a three-dimensional wave-diffusion equation is investigated as a mathematical model capable of describing both oscillatory behaviour and energy dissipation within a unified framework.

What is new? The novelty of this study lies in: deriving analytical solutions of the three-dimensional wave-diffusion equation using the method of separation of variables; establishing the relationship between damping parameters and spectral characteristics of the system; and providing a clear physical interpretation of the obtained solutions in terms of energy dissipation. This approach offers a general analytical model applicable to various physical systems involving coupled wave-diffusion processes.

Mathematical Model. The classical three-dimensional wave equation is given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

where $u(x, y, z, t)$ is the wave potential function, c is the wave propagation speed, ∇ and denotes the Laplace operator.

To account for energy dissipation in real media, a damping (diffusion) term is introduced, leading to the wave-diffusion equation of the form

$$\frac{\partial^2 u}{\partial t^2} + \alpha \frac{\partial u}{\partial t} = c^2 \nabla^2 u + f(x, y, z, t),$$

where $\gamma > 0$ is the damping coefficient and $f(x, y, z, t)$ represents an external source term.

This equation is widely used to describe acoustic wave attenuation, thermoelastic phenomena, electromagnetic signal decay, and oscillatory processes in dissipative media.

Method of Separation of Variables

To obtain an analytical solution, we apply the method of separation of variables by assuming a solution of the form $u(x, y, z, t) = X(x)Y(y)Z(z)T(t)$

Substituting this expression into the governing equation and separating spatial and temporal variables yields

$$\frac{T''(t) + \alpha T'(t)}{c^2 T(t)} = \frac{\nabla^2 (XYZ)}{XYZ} = -\lambda^2,$$

where λ is a spectral parameter.

This leads to the following two independent equations:

Spatial part:

$$\nabla^2 (XYZ) + \lambda^2 XYZ = 0,$$

Temporal part:

$$T''(t) + \alpha T'(t) + c^2 \lambda^2 T(t) = 0.$$

Spectral Problem and Solutions

The spatial equation represents a classical spectral problem for the Laplace operator, where the eigenvalues λ_n and eigenfunctions $X_n(x, y, z)$ are determined by the imposed boundary conditions.

The temporal equation corresponds to a damped harmonic oscillator, whose solution is given by

$$T(t) = e^{-\frac{\alpha t}{2}} \left(C_1 e^{i\omega t} + C_2 e^{-i\omega t} \right), \quad \omega = \sqrt{c^2 \lambda^2 - \frac{\alpha^2}{4}}.$$

The general solution of the wave-diffusion equation can therefore be expressed as a series expansion

$$u(x,y,z,t)=e^{-\frac{\alpha t}{2}}\sum_{n,m,l}A_{nml}\Phi_{nml}(x,y,z)\cos(\omega_{nml}t+\phi_{nml}),$$

Results and Discussion. The obtained analytical solutions reveal the fundamental properties of the three-dimensional wave-diffusion process. In particular, the exponential factor in the temporal solution demonstrates the direct influence of the damping coefficient on energy dissipation. For positive values of the damping coefficient, the wave amplitude decays over time, which is consistent with physical observations in real dissipative media.

The spectral parameters depend on the spatial boundary conditions and reflect the geometric properties of the system. If the condition $c^2\lambda_n > \frac{\gamma^2}{4}$ is satisfied, the system exhibits damped oscillatory behaviour; otherwise, an aperiodic decay occurs. This allows for stability analysis of the model under various physical configurations.

The results confirm that the proposed wave-diffusion model is capable of describing acoustic attenuation, thermoelastic interactions, electromagnetic signal decay, and other dissipative wave phenomena in both qualitative and quantitative terms. Moreover, the analytical solutions provide a solid foundation for further numerical simulations and practical applications.

Conclusion. In this paper, a three-dimensional wave-diffusion model for physical process modelling has been investigated. Analytical solutions were obtained using the method of separation of variables, and their physical interpretation was provided in terms of energy dissipation. The results demonstrate that the damping coefficient plays a crucial role in determining system stability and wave attenuation.

The proposed model offers a unified mathematical framework for describing coupled wave-diffusion phenomena and can be effectively applied to a wide range of physical systems. Future research will focus on numerical methods and complex boundary conditions to extend the applicability of the model.

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